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ECONOMIC ORDER MODEL (nQ,R,T): CONSTANT LEAD TIMES AND EXPONENTIAL BACKORDER COSTS

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Abstract
At each periodic review, the quantity ordered is multiple of Q, nQ and the reorder level is R, The backorder cost $C_\beta(t)$ is b1 e b2 t. The expected backorder cost is derived by obtaining the difference between the expected backorder cost at time t+l and t+l+T.
In this paper demand is assumed to follow a normal distribution.
Some basic mathematics of the properties of a normal distribution is introduced to simplify the derivation of the equations.
The first order derivatives of the backorder costs are given..

Introduction
In this paper the cost depends upon the length of time for which the backorder exists and is taken as an exponential function. backorders that are not met on time incur various types of costs, which could be linear, quadratic, exponential or any other function of the time orders are not met.
Organizations that store thousands of products could face serve inventory costs, when the backorder cost is an exponential function.

Literature Review
The simple models of economic backorder inventory control model (nQ,R,T) model for linear backorder were extensively dealt with by Hadley and Whitin.
(Ref 1)
Uthayakumar and Parrathi (2009) investigates a continuous review inventory model to reduce lead time, yield variability and set up costs simultaneously, through capital investments.
The backorder rate is depending on the lead time and the amount of shortage.
Zhang G.U and Dathwo (2003) develops a hybrid inventory system with a time limit in backorders.

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We derive expected backorder costs where the cost of backorder is

$C_B(t) = e^{b_2}t \qquad b_2 > 0$

When t is the length of time of a backorder

$\sigma^2 z$ is the variance of demand over a period Z

If the inventory position of the system immediately after the review at time t, is

R + Y, the expected backorder cost at time t, is

$$= \frac{1}{Q}\int_0^Q D\int_0^L D\int_0^t \frac{C_B(t-z)}{\sqrt{\sigma^2 t}}\,\mathrm{g}\left(\frac{R+Y-Dz}{\sqrt{\sigma^2 t}}\right) dzdtdY \qquad (1)$$

Similarly the expected backorder costs at time t+L+T

$$= \frac{1}{Q}\int_0^Q D\int_0^{L+T} D\int_0^t \frac{C_B(t-z)}{\sqrt{\sigma^2 t}}\,\mathrm{g}\left(\frac{R+Y-Dz}{\sqrt{\sigma^2 t}}\right) dzdtdY \qquad (2)$$

$$C_B(L-z) = b_1 e^{b_2(L-Z)} \qquad b_2 > 0 \qquad (3)$$

$$Let\ G_1\,(Q,R,L) = \frac{1}{Q}\int_0^Q D\int_0^L D\int_0^t \frac{C_B(t-z)}{\sqrt{\sigma^2 t}}\,\mathrm{g}\left(\frac{R+Y-Dz}{\sqrt{\sigma^2 L}}\right) dzdtdY \qquad (4)$$

Substitute for $C_B(t-z)\ in\ (4)$

We have

$$G_1(Q,R,L) = \frac{1}{Q}\int_0^Q\int_0^L D\int_0^t \frac{b_1 esp\big(b_2(t-z)\big)}{\sigma^2 t}\,\mathrm{g}\left(\frac{R+Y-DZ}{\sqrt{\sigma^2 L}}\right) dzdtdY \qquad (5)$$

$$Let\ G_2(R+Y,L) = \frac{D}{\sqrt{\sigma^2 L}}\int_0^L b_1\, esp(b_2(t-z)\mathrm{g}\left(\frac{R+Y-DZ}{\sqrt{\sigma^2 L}}\right) dzdtdY$$

$$= b_1 e^{b_2 L}\exp\left(\frac{\sigma^2 b_2 L}{2D^2} - b_2\frac{(R+Y)}{D}\right) F\left(\frac{R+Y-\frac{\sigma^2 b_2 L}{D}-DL}{\sqrt{\sigma^2 L}}\right) \qquad (6)$$

$$Let\ G_3(Q,R) = \int_0^L b_1 esp\,(R+Y,T)dY \qquad (7)$$

Substitute for G_2

$$G_3(Q,R) = b_1 esp\,(b_2 L)\int_0^Q exp\left(\frac{\sigma^2 b_2 L}{2D^2} - b_2\frac{(R+Y)}{D}\right) F\left(\frac{R+Y-\frac{\sigma^2 b_2 L}{D}-DL}{\sqrt{\sigma^2 L}}\right) dY \qquad (8)$$

$$Let\ v = \frac{R+Y-\frac{\sigma^2 b_2 L}{D}-DL}{\sqrt{\sigma^2 L}} \qquad (9)$$

Then

$$G_3(Q,R) = \sqrt{\sigma^2}Lb_1 e^{b_2 L}\int_{\left(R-\frac{\sigma^2 b_2 L}{D}-\frac{DL}{\sqrt{\sigma^2 L}}\right)}^{\left(R+Q-\frac{\sigma^2 b_2 L}{D}-DL\right)/\sqrt{\sigma^2 L}} \frac{exp}{2D^2}\sigma^2 b_2^2 L\ exp - \frac{b_2}{D}\left(\sqrt{\sigma^2}LV + \frac{\sigma^2 L b_2}{D} + DL\right) F(v)dv$$

Simplifying we have

$G_3(Q,R)$

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$$= b_1\sqrt{\sigma^2 L}\exp\frac{-\sigma^2 b_2 L}{2D^2}\int_{\left(R-\frac{\sigma^2 b_2 L}{D}-DL\right)/\sqrt{\sigma^2 L}}^{\left(R+Q-\frac{\sigma b_2 L}{D}-DL\right)/\sqrt{\sigma^2 L}} exp - \frac{b_2\sqrt{\sigma^2 L}}{D}.vF(V)dv$$

Integrating by parts we have

$$G_3(Q,R) = b_1\sqrt{\sigma^2 L}\, exp\frac{-\sigma^2 b_2 L}{2D^2}\left[\frac{-D}{b_2}\sqrt{\sigma^2 L}\, exp\frac{-b_2\sqrt{\sigma^2 L}}{D}VF(V)\right]_{\frac{\left(R-\frac{\sigma^2 b_2 L}{D}-DL\right)}{\sqrt{\sigma^2 L}}}^{\frac{\left(R+Q-\frac{\sigma b_2 L}{D}-DL\right)}{\sqrt{\sigma^2 L}}}$$

$$\frac{-Db_1}{b_2}\frac{1}{\sqrt{2\pi}}exp\frac{-\sigma^2 b_2 L}{2D^2}\int_{\left(R-\frac{\sigma^2 b_2 L}{D}-DL\right)/\sqrt{\sigma^2 L}}^{\left(R+Q-\frac{\sigma b_2 L}{D}-DL\right)/\sqrt{\sigma^2 L}} exp\left[\frac{-b_2 V\sqrt{\sigma^2 L}}{D}-\frac{V^2}{2}\right]dv$$

Nothing that

$$\frac{-\sigma^2 b_2 L}{2D^2}-\frac{b_2 V\sqrt{\sigma^2 L}}{D}-\frac{V^2}{2}$$

$$= -\frac{1}{2}\left(V^2+2\frac{b_2 v\sqrt{\sigma^2 L}}{D}+\frac{\sigma^2 b_2 L}{2D^2}\right)$$

$$= \frac{-1}{2}\left(V+b_2\frac{\sqrt{\sigma^2 L}}{D}\right)^2$$

Then $G_3(Q,R) = \frac{-Db_1}{b_2}\left[\exp\left[\frac{-\sigma^2 b_2 L}{2D^2}-\frac{b_2\sqrt{\sigma^2 L}}{D}V\right]F(v)\right]_{\left(R-\frac{b_2\sigma^2 L}{D}-DL\right)v\sqrt{\sigma^2 L}}^{\frac{\left(R+Q-\frac{\sigma b_2 L}{D}-DL\right)}{\sqrt{\sigma^2 L}}}$

$$\frac{-Db_1}{b_2\sqrt{2\pi}}\int_{\frac{(R-\sigma^2 b_2 l/D-DL)}{\sqrt{\sigma^2 L}}}^{\left(R+Q-\frac{\sigma^2 b_2 L}{D}-DL\right)/\sqrt{\sigma^2 L}} exp-\frac{1}{2}\left(V+b_2\frac{\sqrt{\sigma^2 L}}{D}\right)^2 dv$$

$Let\ x = v+\frac{b_2\sqrt{\sigma^2 L}}{D} and\ substitute\ into\ integral$

Then $G_3(Q,R) = \frac{-Db_1}{b_2}\left[\exp-\left(\frac{\sigma^2 b_2 L}{2D^2}+\frac{b_2\sqrt{\sigma^2 L}}{D}\right)F(V)\right]_{\left(R-\frac{\sigma^2 b_2 L}{D}-DL\right)/\sqrt{\sigma^2 L}}^{\left(R+Q-\frac{\sigma^2 b_2 L}{D}-DL\right)/\sqrt{\sigma^2 L}}$

$$\frac{-Db_1}{b_2\sqrt{2\pi}}\int_{\frac{R-DL}{\sqrt{\sigma^2 L}}}^{\frac{R+Q-DL}{\sqrt{\sigma^2 L}}} esp-\frac{x^2}{2}dx$$

Integrating we have

$$G_3(Q,R) = \frac{-Db_2}{b_2}\left[\exp-\left(\frac{\sigma^2 b_2 L}{2D^2} esp\frac{b_2 V\sqrt{\sigma^2 L}}{D}\right)F(V)\right]_{\left(R-\frac{\sigma^2 b_2 L}{D}-DL\right)/\sqrt{\sigma^2 L}}^{\left(R+Q-\frac{\sigma^2 b_2 L}{D}-DL\right)/\sqrt{\sigma^2 L}} + \frac{-Db_1}{b_2}[F(V)]_{(R-DL)/\sqrt{\sigma^2 L}}^{(R+Q-DL)/\sqrt{\sigma^2 L}}$$

Expanding

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$$G_3(Q,R) = \frac{Db_1}{b_2}\left[exp - \left(\frac{\sigma^2 Lb_2}{2D^2}\right) exp\,\frac{b_2}{D}\left(R - \frac{\sigma^2 Lb_2}{D} - DL\right) F\left(\frac{R - \frac{\sigma^2 b_2 L}{D} - DL}{\sqrt{\sigma^2 L}}\right) - \exp\left(\frac{\sigma^2 b_2 L}{2D^2} + \frac{b_2}{D}\left(R + Q - \frac{\sigma^2 b_2 L}{D} - DL\right)\right) F\left(\frac{R+Q-\frac{\sigma^2 b_2 L}{D} - DL}{\sqrt{\sigma^2 L}}\right)\right]$$

$$-\frac{Db_1}{b_2}\left(F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right) - F\left(\frac{R+Q-DL}{\sqrt{\sigma^2 L}}\right)\right) \quad (10)$$

Integrating

G_1 (Q, R, L) with respect to Q and substituting for G_3(Q, R) in equation (5)

$$G_1(Q,R,L) = \frac{D}{Q}\int_0^L \frac{b_1}{b_2}\left(exp - \left(\frac{b_2 R}{D} - b_4 t - \frac{\sigma^2 t b_2^2}{2D^2}\right) F\left(\frac{R - \frac{\sigma^2 b_2 t}{D} - Dt}{\sqrt{\sigma^2 t}}\right) - F\left(\frac{R-Dt}{\sqrt{\sigma^2 t}}\right)\right) dt$$

$$-D\int_0^L \frac{b_1}{b_2}\left(exp - \left(\frac{b_2(R+Q)}{D} - b_2 t - \frac{\sigma^2 t b^2}{2D^2}\right) F\left(\frac{R+Q-\frac{\sigma^2 t b_2}{D} - Dt}{\sqrt{\sigma^2 t}}\right) - F\left(\frac{R+Q-Dt}{\sqrt{\sigma^2 t}}\right)\right) dt \quad (11)$$

Let

$$G_4(R,L) = \int_0^L \frac{b_1}{b_2}\left(exp - \left(\frac{b_2 R}{D} - b_2 t - \frac{\sigma^2 t b_2^2}{2D^2}\right) F\left(\frac{R - \frac{\sigma^2 b_2 t}{D} - Dt}{\sqrt{\sigma^2 t}}\right) - F\left(\frac{R-Dt}{\sqrt{\sigma^2 t}}\right)\right) dt \quad (12)$$

Thus

$$G_1(Q,R,L) = \left(G_4(R,L) - G_4(R+Q,L)\right)/Q \quad (13)$$

Rearranging the exponential terms of equation (12)

$$G_4(R,L) = D\int_0^L \frac{b_1}{b_2} \exp\left[t\,\frac{\sigma^2 b_2^2}{2D^2} + b_2\right) - \frac{b_2 R}{D}\right] F\left(R - \frac{\frac{\sigma^2 b_2 t}{D}}{\sqrt{\sigma^2 t}} - Dt\right) dt$$

Integrating by parts

$$\frac{1}{D} G_4(R,L) = \frac{b_1}{b_2}\left(\frac{2D^2}{\sigma^2 b_2^2 + 2D^2 b_2}\right)\left[\exp\left(\frac{\sigma^2 b_2^2}{2D^2} + b_2\right) - \frac{b_2 R}{D}\right) * F\left(\frac{R - \frac{\sigma^2 t b_2}{D} - DT}{\sqrt{\sigma^2 t}}\right)\Bigg]_0^L$$

$$-\frac{b_1}{2b_2}\left(\frac{2D^2}{\sigma^2 b_2 + 2D^2 b_2}\right)\int_0^L exp\left(t\left(\frac{\sigma^2 b_2^2}{2D^2} + b_2\right) - \frac{b_2 R}{D}\right)$$

$$exp - \frac{1}{2}\left(\frac{R - \frac{\sigma^2 b_2 t}{D} - Dt}{\sqrt{\sigma^2 t}}\right)^2 \left(\frac{D^2 + \sigma^2 b_2}{D\sqrt{\sigma^2 t}} + \frac{R}{\sigma t^{3}/_{2}}\right) dt - \frac{b_2}{b_2}\int_0^L F\left(\frac{R-Dt}{\sqrt{\sigma^2 t}}\right) dt \quad (15a)$$

Nothing that

$$t\left(\frac{\sigma^2 b_2^2}{2D^2} + b_2\right) - \frac{b_2 R}{D} - \frac{1}{2\sigma^2 t}\left(R - \frac{\sigma^2 b_2 t}{D} - Dt\right)^2$$

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$$= \frac{-1}{2\sigma^2 t}(R^2 - 2DRt + D^2t^2) = -\frac{1}{2}\left(\frac{R-Dt}{\sqrt{\sigma^2 t}}\right)^2$$

Hence simplifying

$$\frac{1}{D}G_4(R,L) = \frac{b_1}{b_2}\left(\frac{2D^2}{\sigma^2 b_2^2 + 2D^2 b_2}\right)\left[\exp t\left(\frac{\sigma^2 b_2^2 + 2D^2 b_2}{2D^2}\right) - \frac{b_2 R}{D} F\left(\frac{R - \frac{\sigma^2 b_2 t}{D} - Dt}{\sqrt{\sigma^2 t}}\right)\right]_0^L$$

$$-\frac{b_1}{b_2}\left(\frac{2D^2}{\sigma^2 b_2^2 + 2D^2 b_2}\right)\int_0^L \left(\frac{D^2 + \sigma^2 b_2}{2D\sqrt{\sigma^2 t}} + \frac{R}{2\sigma t^{3}/_{2}}\right)\frac{1}{2\pi} exp - \frac{1}{2}\left(\frac{R-Dt}{\sqrt{\sigma^2 t}}\right)^2 dt - \frac{b_1}{b_2 Q} F\left(\frac{R-Dt}{\sqrt{\sigma^2 t}}\right) dt \qquad (15b)$$

BASIC MATHEMATICS

The basic mathematics equations are well developed in Hadley and Whitin Ref (1)

Let $Z_n(x,T) = \int_0^T t^n \, g(x, Dt) dt$

And $R_n(x,T) = \int_0^T t^n \, F(x, Dt) dt$

$$Z_{n+1}(x,T) = \int_0^T t^{n+1} \, g(x, Dt) dt$$

$$henceZ_{n+1}(x,T) = -\frac{2\sigma^2 T^{n+1}}{D^2\sqrt{2\pi\sigma^2 T}} exp - \frac{1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 - \frac{\sigma^2(2n+1)}{2D^2} Z_n(x,T) + \frac{x^2}{D^2} Z(x,T)$$

Hence

$$Z_1(x,T) = \frac{2\sigma^2 T}{D^2\sqrt{2x\sigma^2 T}} exp - \frac{1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 + \frac{\sigma^2}{D^2} Z_0(x,T) + \frac{x^2}{D^2} Z(x,T) \qquad (16)$$

Hence $\frac{x^2}{D^2} Z_{-1}(x,T) = Z_1(x,T) + \frac{2\sigma^2 T}{D^2\sqrt{2\pi\sigma^2 T}} exp - \frac{1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 - \frac{\sigma^2}{D^2} Z_0(x,T)$

$$Z_0(x,T) = \int_0^T \frac{1}{\sqrt{2\pi\sigma^2 t}} exp \frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 dt$$

$$= \int_0^T \left(\frac{t^{\frac{1}{2}}}{\sqrt{2\pi\sigma^2}}\right) exp - \frac{1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right) dt$$

Integrating by parts and applying

$$\frac{d\left(exp - \frac{1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2\right.}{dt} = \left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)\left(\frac{D}{\sigma^2 t} + \frac{(x-Dt)}{2\sigma t^{3}/_{2}}\right) exp - \frac{1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2$$

$$Z_0(x,T) = \left[\frac{2t^{\frac{1}{2}}}{\sqrt{2\pi\sigma^2}} exp - \frac{1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2\right]_0^T - 2\int_0^T \frac{t^{\frac{1}{2}}}{\sqrt{2\pi\sigma^2}}\left(\frac{(x-Dt)D}{\sigma^2 t} + \frac{(x-Dt)^2}{2\sigma^2 t^2}\right)$$

$$exp - \frac{1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 dt \qquad (17)$$

Simplifying

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$$Z_0(x,T) = \frac{2t^{\frac{1}{2}}}{\sqrt{2\pi\sigma^2}} exp - \frac{1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2\Bigg]_0^T + \frac{D^2}{\sigma^2}\int_0^T \frac{t}{\sqrt{2\pi\sigma^2}} exp - \frac{1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 dt$$

$$-\frac{x}{\sigma^2}\int_0^T \frac{t^{-3/2}}{\sqrt{2\pi\sigma^2}} exp - \frac{1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 dt$$

Let $Z_1(x,T) = \int_0^T \frac{t}{\sqrt{2\pi\sigma^2}} exp - \frac{1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 dt$ (18)

$$Z_0(x,T) = \int_0^T g\,(x, Dt) dt$$

$$= \frac{1}{D} F\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right) - \exp\left(\frac{2Dx}{\sigma^2}\right) F\left(\frac{x+Dt}{\sqrt{\sigma^2 t}}\right) \quad (19)$$

Hence differentiating with respect to x

$$\frac{\partial Z_0(x,T)}{\partial x} = -exp - \frac{1}{2}\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right)^2 - \frac{2D}{\sigma^2}\exp\left(\frac{2Dx}{\sigma^2}\right) F\left(\frac{x+DT}{\sqrt{\sigma^2 T}}\right) + \frac{1}{D\sqrt{2\pi\sigma^2 T}} exp - \frac{1}{2}\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right)^2$$

Hence substituting into $Z_0(x,T)$ and $\partial Z_0(x,T)$ and simplifying we have

$$Z_1(x,T) = \frac{-2\sqrt{\sigma^2 T}}{D^2} exp - \frac{1}{2}\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right)^2 + \frac{\sigma^2}{D^3}\left(1 + \frac{Dx}{\sigma^2}\right) F\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right) + \frac{1}{D_2}\left(x - \frac{\sigma^2}{D}\right) exp\,\frac{2Dx}{\sigma^2} F\left(\frac{x+Dt}{\sqrt{\sigma^2 T}}\right) \quad (20)$$

Hence substituting $Z_1\ (x,T)$ into $Z_0\ (x,T)$

$$Z_0(x,T) = \frac{2t^{\frac{1}{2}}}{\sqrt{2\pi\sigma^2}} exp - \frac{1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2\Bigg]_0^T + \frac{D^2}{\sigma^2}\int_0^T \frac{t^{-3/2}}{\sqrt{2\pi\sigma^2}} exp\,\frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 dt$$

Nothing that

$$\frac{\partial Z_0(x,T)}{\partial x} = \frac{-1}{2\pi}\int_0^T \left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right) exp\,\frac{-1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 dt$$

Hence substituting into Z_0 (x, T) and simplifying

$$Z_0(x,T) = \frac{D^2}{\sigma^2} Z_1(x,T) + x\partial\frac{Z_0}{\partial x}(x,T) - \frac{DxZ_0(x,T)}{\sigma^2} + \left[\frac{2t^{\frac{1}{2}}}{\sqrt{2\pi\sigma^2}} exp - \frac{1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2\right]_0^T$$

Hence

$$Z_1(x,T) = \frac{\sigma^2}{D^2}\left[-\frac{2t^{\frac{1}{2}}}{\sqrt{2\pi\sigma^2}} exp - \frac{1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2\right]_0^T + \frac{\sigma^2}{D^2}\left(1 + \frac{Dx}{\sigma^2}\right) Z_0(x,T) - \frac{\sigma_x^2}{D^2}\partial\frac{Z_0(x,T)}{\partial x}$$

$R_n(x,T)$ is defined as

$$R_n(x,T) = \int_0^T t^n F\left(\frac{x-Dt}{\sqrt{\sigma^2 T}}\right) dt \qquad n = 0,1,2 \ldots\ldots\ldots\ldots\ldots \quad (21)$$

Solving (21) by integrating by parts

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$$R_n(x,T)=\frac{t^{n+1}}{n+1}F\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)\Big]_0^T-\frac{1}{\sqrt{2\pi}}\int_0^T\frac{t^{n+1}}{(n+1)\sqrt{\sigma^2 t}}\left(\frac{D}{\sqrt{\sigma^2 t}}+\frac{(x-Dt)}{2\sigma t^{3}/_2}\right)exp-\frac{1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)^2 dt$$

Hence substituting in $Z_{n+1}(x,T) and\ Z_n(x,T) we\ have$ (22)

$$R_n(x,T)=\frac{T^{n+1}}{n+1}F\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right)-\frac{D}{2(n+1)}Z_{n+1}(x,T)-\frac{x}{2(n+1)}Z_n(x,T) \qquad n=0,1,\ldots\ldots \tag{22}$$

Thus $R_0(x,T)=TF\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right)-\frac{D}{2}Z_1(x,T)-\frac{x}{2}Z_0(x,T)$ (23)

Substituting for

$Z_0(x,T), from$ 19 $and\ Z_1(x,T) from$ (20) $and\ R_0(x,T) from$ (23)$in\ G_4(x,T) in$ (15)

We have

$$\frac{1}{D}G_4(x,T)=\frac{b_1}{b_2}\left(\frac{2D^2}{\sigma^2 b_2+2\sigma^2 b_2}x,T\right)\left[\exp\left(\frac{\sigma^2 b_2+2\sigma^2 b_2}{2D^2}\right)-\frac{b_2 R}{D}F\left(\frac{x-\frac{t(\sigma^2 b_2-D^2)}{D}}{\sqrt{\sigma^2 t}}\right)\right]_0^T$$

$$-\frac{b_1}{b_2}\left(\frac{2D^2}{\sigma^2 b_2+2\sigma^2 b_2}\right)\left(\frac{D^2+\sigma^2 b_2}{2D^2}\right)$$

$$\left[F\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right)+exp\frac{2Dx}{\sigma^2}F\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right)-\frac{b_1}{2b_2}\left(\frac{2D^2}{\sigma^2 b_2+2\sigma^2 b_2}\right)\left(F\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right)+esp\frac{2Dx}{\sigma^2}F\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right)\right)\right.$$

$$-\frac{b_1}{b_2}\left(\left(T-\frac{x}{D}-\frac{\sigma^2}{2D^2}\right)F\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right)*\frac{2\sigma^2 T}{2\sqrt{2\pi\sigma^2}}exp-\frac{1}{2}\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right)+\frac{\sigma^2}{2D^2}esp\frac{2Dx}{\sigma^2}F\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right)\right) \tag{24}$$

Rearranging terms

$$\frac{1}{D}G_4(x,T)=\frac{2b_1 D^2}{\sigma^2 b_2^3+2D^2 b_2^2}\exp\quad T\left(\frac{\sigma^2 b_2^2+2D^2 b_2}{2D^2}\right)exp-\frac{bx}{D}F\left(\frac{X-\left(\frac{\sigma^2 b_2}{D}+D\right)}{\sqrt{\sigma^2 t}}\right)$$

$$\frac{b_1}{b_2}\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right)\left[-\left(\frac{2D^2}{\sigma^2 b_2+2\sigma^2 b_2}\right)\left(\frac{D^2+\sigma^2 b_2}{2D^2}\right)-\frac{1}{2}\left(\frac{2D^2}{\sigma^2 b_2+2\sigma^2 b_2}\right)-\left(T-\frac{x}{D}-\frac{\sigma^2}{2D^2}\right)\right]+$$

$$esp\frac{2Dx}{\sigma^2}F\left(\frac{X-DT}{\sqrt{\sigma^2 T}}\right)\left[\frac{b_1}{b_2}\left(\frac{2D^2}{\sigma^2 b_2+2\sigma^2 b_2}\right)\left(\frac{D^2+\sigma^2 b_2}{2D^2}\right)-\left(\frac{2D^2}{\sigma^2 b_2+2\sigma^2 b_2}\right)\frac{b_1}{2b_2}-\frac{\sigma^2}{2D^2}\frac{b_1}{b_2}-\frac{2\sqrt{\sigma^2 t}}{Db_2}g\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right)\right.$$

Simplifying

$$G_4(X,T)$$

$$=\frac{2D^3 b_1}{(\sigma^2 b_2^3+2D^2 b_2^2)}expT\left(\frac{\sigma^2 b_2^2+2D^2 b_2}{2D^2}\right)exp\frac{-b_2}{D}F\left(\frac{R-DT\left(1+\frac{\sigma^2 b_2}{D}\right)}{\sqrt{\sigma^2 T}}\right)$$

$$-\frac{b_1}{b_2}F\left(\frac{R-DT}{\sqrt{\sigma^2 T}}\right)\left[DT-X-\frac{\sigma^2}{2D}+\frac{D}{b_2}\right]-\frac{\sigma^4 b_1 b_2^2}{2D(\sigma^2 b_2^2+2D^2 b_2)b_2}\exp\frac{2XD}{\sigma^2}F\left(\frac{X-DT}{\sqrt{\sigma^2 T}}\right)$$

$$-\frac{2\sqrt{\sigma^2 t}}{b_2}b_1 g\left(\frac{X-DT}{\sqrt{\sigma^2 T}}\right) \tag{25}$$

From (13)

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G_1 (Q,R,L) = (G_4(R,L)-G_4(R+Q,L))/Q
From (2)
Expected backorder cost at T+L, averaging over the states of y

$$= {}^{1}/_{Q}\ (G_4(R,L) - G_4(R+Q,L)) \qquad (26)$$

And the expected backorder cost at t+L+T averaging over the states of y

$$= {}^{1}/_{Q}\ (G_4(R,L+T) - G_4(R+Q,L+T)) \qquad (27)$$

Hence the expected backorder cost per year
G_5(Q,R,T)

$$= {}^{1}/_{Q}\ (G_4(R,L+T) - G_4(R,L) - G_4(R+Q,L+T) + G_4\ (R+Q,L)) \qquad (28)$$

The traditional first derivatives of C are the derivatives of G_4 (R, Z)
With respect to R and Ƶ
Differentiating with respect to R

$$\frac{\partial G_4(R-,z)}{\partial R} = \frac{-2D^2 b_1}{(\sigma^2 b_2^3 + 2D^2 b_2^2)} exp\left(\frac{\sigma^2 b_2^2 + 2D^2 b_2}{2D^2} - b_2 R\right)$$

$$F\left(\frac{R-z\left(D+\frac{\sigma^2 b_2}{D}\right)}{\sqrt{\sigma^2 T}}\right) - \frac{2D^3 b_1}{(\sigma^2 b_2^3 + 2D^2 b_2^2)} exp - \frac{1}{2}\left(\frac{R-z\left(D+\frac{\sigma^2 b_2}{D}\right)}{\sqrt{\sigma^2 T}}\right)^2$$

$$exp\left(\frac{\sigma^2 b_2^2 + 2D^2 b_2^2 z}{2D^2} - \frac{b_2 R}{D}\right) - Db_1 exp - \frac{1}{2}\left(\frac{R-Dz}{\sqrt{\sigma^2 z}}\right)^2 \left[\frac{(R-Dz)}{D} - \frac{1}{b_2} + \frac{\sigma^2}{2D_2^2}\right] + \frac{b_1}{b_2} F\left(\frac{R-Dz}{\sqrt{\sigma^2 z}}\right)$$

$$-\frac{2D^2}{\sigma^2}\frac{\sigma^4 b_1 b_2}{2D^2(\sigma^2 b_2^2 + 2D^2 b_2) b_2} exp\frac{2DR}{\sigma^2} F\left(\frac{R-Dz}{\sqrt{\sigma^2 z}}\right) + \frac{2b_1}{b_2}\left(\frac{R-Dz}{\sqrt{\sigma^2 z}}\right) exp\frac{\frac{-1}{2}}{\sqrt{2\pi\sigma^2 z}}\left(\frac{R-Dz}{\sqrt{\sigma^2 z}}\right)^2 / \sqrt{2\pi\sigma^2 z}$$

$$+\frac{\sigma^4 b_1 b_2}{2D^2(\sigma^2 b_2^2 + 2D^2 b_2) b_2} exp - \frac{1}{2}\left(\frac{R-D_2}{\sqrt{\sigma^2 z}}\right)^2 / \sqrt{2\pi\sigma^2 z}$$

Simplifying

$$\frac{\partial G_4(R,z)}{\partial R} = \frac{-2D^2 b_1}{(\sigma^2 b_2^2 + 2D^2 b_2)} \exp\left[z\left(\frac{\sigma^2 b_2^2 + 2D^2 b_2}{2D^2}\right) - b_2 R\right]$$

$$F\left(\frac{R-z\left(D+\frac{\sigma^2 b_2}{D}\right)}{\sqrt{\sigma^2 T}}\right) + \frac{b_1}{b_2}(R-Dz) exp - \frac{1}{2}\left(\frac{R-D_2}{\sqrt{\sigma^2 z}}\right)^2 / \sqrt{2\pi\sigma^2 z}$$

$$\frac{b_1}{b_2} F\left(\frac{R+DƵ}{\sqrt{\sigma^2 z}}\right) - \frac{\sigma^2 b_2^2 b_1}{b_2(\sigma^2 b_2^2 + 2D^2 b_2)} exp\frac{2DR}{\sigma^2} F\left(\frac{R+DƵ}{\sqrt{\sigma^2 z}}\right)$$

Differentiating with respect to Z

$$\frac{\partial G_4(R,Ƶ)}{\partial Z} = \frac{Db_1}{b_4} exp - \left(\frac{b_2 R}{D} - b_2 z - \frac{\sigma^2 z b_2^2}{2D^2}\right)$$

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$$F\left(\frac{R-\frac{\sigma^2 b_2^2}{D}-DL}{\sqrt{\sigma^2 z}}\right)$$

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